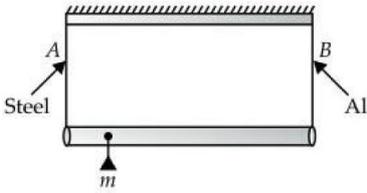


TOPICS : Mechanics of Solids

- Force constant of a spring (k) is analogous to
 - $\frac{YA}{L}$
 - $\frac{YL}{A}$
 - $\frac{AL}{Y}$
 - ALY
- The Young's modulus of brass and steel are respectively $1.0 \times 10^{11} \text{ N m}^{-2}$ and $2.0 \times 10^{11} \text{ N m}^{-2}$. A brass wire and a steel wire of the same length are extended by 1 mm each under the same force. If radii of brass and steel wires are R_B and R_S respectively, then
 - $R_S = \sqrt{2} R_B$
 - $R_S = \frac{R_B}{\sqrt{2}}$
 - $R_S = 4R_B$
 - $R_S = \frac{R_B}{2}$
- Modulus of rigidity of ideal liquids is
 - infinity
 - zero
 - unity
 - some finite small non-zero constant value
- Which of the statement is incorrect ?
 - Steel is more elastic than rubber.
 - The stretching of a coil is determined by its shear modulus
 - Stress is a vector quantity
 - Metal have larger values of Young's modulus than alloys and elastomers
- The ratio of tensile stress to the longitudinal strain is defined as
 - bulk modulus
 - Young's modulus
 - shear modulus
 - compressibility
- The average depth of Indian ocean is about 3000 m. The fractional compression, $\frac{\Delta V}{V}$ of water at the bottom of the ocean is
(Given : Bulk modulus of the water = $2.2 \times 10^9 \text{ N m}^{-2}$ and $g = 10 \text{ ms}^{-2}$)
 - 0.82%
 - 0.91%
 - 1.36%
 - 1.24%
- A structural steel rod has a radius of 10 mm and a length of 1 m. A 100 kN force stretches it along its length. The strain on the rod is
($Y_{\text{steel}} = 200 \times 10^9 \text{ N m}^{-2}$)
 - 1.6 mm
 - 2.6 mm
 - 3.6 mm
 - 4.6 mm
- A wire of length L and radius r is clamped at one end. On stretching the other end of the wire with a force F , the increase in its length is l . If another wire of same material but of length $2L$ and radius $2r$ is stretched with a force $2F$, the increase in its length will be
 - $\frac{l}{4}$
 - $\frac{l}{2}$
 - l
 - $2l$
- Which of the following substances has highest value of Young's modulus ?
 - Aluminium
 - Iron
 - Copper
 - Steel
- Which one of the following statements are wrong ?
 - Young's modulus for a perfectly rigid body is zero
 - Bulk modulus is relevant for solids, liquids and gases
 - The young's modulus and shear modulus are relevant for solids
 - The stretching of a coil spring is determined by its shear modulus
- A rod of length L and negligible mass is suspended at its two ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in the figure. The cross-sectional areas of wires A and B are 1 mm^2 and 2 mm^2 respectively.
($Y_{\text{Al}} = 70 \times 10^9 \text{ N m}^{-2}$ and $Y_{\text{steel}} = 200 \times 10^9 \text{ N m}^{-2}$)



To have equal stress in both the wires, mass m should be suspended at a distance of

 - $\frac{1}{3}L$ from the wire A
 - $\frac{1}{2}L$ from the wire B
 - $\frac{2}{3}L$ from the wire B
 - $\frac{2}{3}L$ from the wire A

TOPICS : Mechanics of Solids (SOLUTION)

1. (a) : $Y = \frac{FL}{A\Delta L}$ or $F = \left(\frac{YA}{L}\right)\Delta L$
Comparing this with $F = k\Delta x$, we get $k = \frac{YA}{L}$

2. (b) : Increase in length
$$\Delta L = \frac{FL}{YA} = \frac{FL}{Y\pi R^2}$$
As F, L and ΔL are same hence,
 $YR^2 = \text{a constant}$
 $\therefore 2.0 \times 10^{11} R_S^2 = 1.0 \times 10^{11} R_B^2 \Rightarrow R_S = \frac{R_B}{\sqrt{2}}$

3. (b) : Modulus of rigidity of ideal liquids is zero.

4. (c) : Stress is not a vector quantity as it cannot be assigned a specific direction. It is a tensor quantity.

5. (b) : Young's modulus is defined as the ratio of tensile (or compressive) stress to the longitudinal strain.

6. (c) : The pressure exerted by a 3000 m column of water on the bottom layer is
$$P = h\rho g$$
$$= 3000 \text{ m} \times 1000 \text{ kg m}^{-3} \times 10 \text{ m s}^{-2}$$
$$= 3 \times 10^7 \text{ N m}^{-2}$$

Fractional compression $\frac{\Delta V}{V}$ is

$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{3 \times 10^7 \text{ N m}^{-2}}{2.2 \times 10^9 \text{ N m}^{-2}} = 1.36 \times 10^{-2} = 1.36\%$$

7. (a) : Here, $r = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$
 $L = 1 \text{ m}$
 $F = 100 \text{ kN} = 100 \times 10^3 \text{ N} = 10^5 \text{ N}$

Stress on the rod is given by

$$\text{Stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$
$$= \frac{100 \times 10^3 \text{ N}}{3.14 \times (10^{-2} \text{ m})^2} = 3.18 \times 10^8 \text{ N m}^{-2}$$

Elongation,

$$\Delta L = \frac{(F/A)L}{Y} = \frac{(3.18 \times 10^8 \text{ N m}^{-2})(1 \text{ m})}{2 \times 10^{11} \text{ N m}^{-2}}$$
$$= 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm}$$

The strain on the rod is given by

$$\text{Strain} = \frac{\Delta L}{L}$$
$$= \frac{1.59 \times 10^{-3} \text{ m}}{1 \text{ m}} = 1.59 \times 10^{-3} = 1.59 \text{ mm}$$
$$= 1.6 \text{ mm}$$

8. (c) : Let Y be the Young's modulus of the material of the wire. Then
For the first wire

$$Y = \frac{F/\pi r^2}{l/L} = \frac{FL}{\pi r^2 l} \quad \dots(i)$$

As both the wires are made of the same material, so their Young modulus is same.

Let the extension produced in second wire be l' . Then

$$Y = \frac{2F/\pi(2r)^2}{l'/2L} = \frac{FL}{\pi r^2 l'} \quad \dots(ii)$$

Equating (i) and (ii), we get

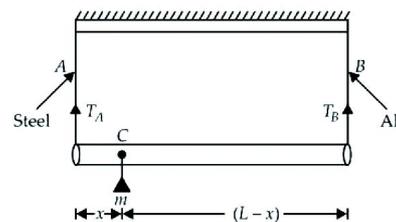
$$\frac{FL}{\pi r^2 l} = \frac{FL}{\pi r^2 l'} \text{ or } l' = l$$

9. (d) :

Substance	Young's modulus 10^9 N m^{-2}
Aluminium	70
Copper	120
Iron	190
Steel	200

From the table, it is clear that among the given substances steel has the highest value of Young's modulus.

10. (d) : All other statements are correct.



Let mass m be suspended at a distance x from the wire A . Let T_A and T_B be tensions in the wire A (steel) and wire B (aluminium) respectively.

$$\therefore \text{Stress in wire } A = \frac{T_A}{A_A}$$

$$\text{Stress in wire } B = \frac{T_B}{A_B}$$

For equal stress in both the wires,

$$\frac{T_A}{A_A} = \frac{T_B}{A_B}$$
$$\frac{T_A}{T_B} = \frac{A_A}{A_B} = \frac{1 \text{ mm}^2}{2 \text{ mm}^2} = \frac{1}{2} \quad \dots(i)$$

As the system is in equilibrium, taking moments about C, we get

$$T_A x = T_B(L - x)$$
$$\frac{L - x}{x} = \frac{T_A}{T_B}$$
$$\frac{L - x}{x} = \frac{1}{2} \quad \text{(Using (i))}$$
$$2L - 2x = x \quad \text{or} \quad x = \frac{2}{3}L$$